The CWENO reconstruction procedure

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Shark-FV 2016

Finite volume schemes

- Method of lines
$$\frac{\mathrm{d}}{\mathrm{d}t}\overline{u}^{n+1}(t) = -\frac{\Delta t}{\Delta \mathrm{x}_j}\left(F_{j+1/2}(t) - F_{j-1/2}(t)\right)$$
- (Approximate) Riemann solver
$$F_{j+1/2}(t) = \mathcal{F}\left(u^-_{j+1/2}(t), u^+_{j+1/2}(t)\right)$$
- Reconstruction procedure
$$u^-_{j+1/2}(t) = \mathcal{R}^-\left(\overline{u}_{j-r}(t), \dots, \overline{u}_{j+r}(t)\right)$$
$$u^+_{j+1/2}(t) = \mathcal{R}^-\left(\overline{u}_{j-r+1}(t), \dots, \overline{u}_{j+r+1}(t)\right)$$

Requirements for a reconstruction procedure on h-adaptive meshes

Must be:

- high-order accurate
- non-oscillatory

For the ADER-DG predictor:

- point values are not enough, but we really need a polynomial defined in the whole cell Ω_j

For high order finite volume methods, it should also be efficient at:

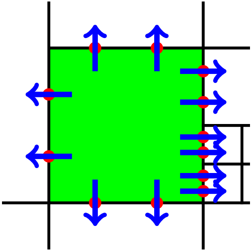
- reconstructing point values at many locations on $\partial\Omega_j$
  Mesh topology ($\Rightarrow$ quadrature nodes) is changing in time
- reconstructing point values at locations inside Ω_j
  for SWE or for entropy indicator
- computing sub-cell averages in refinement

Reconstruction points

For fluxes	For source quadrature
For refinement	For wb quadrature in 1D

Reconstruction points

For fluxes



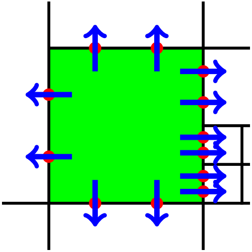
For source quadrature

For refinement

For wb quadrature in 1D

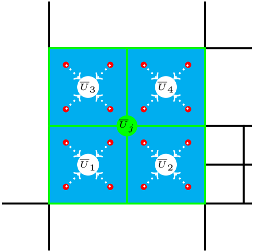
Reconstruction points

For fluxes



For source quadrature

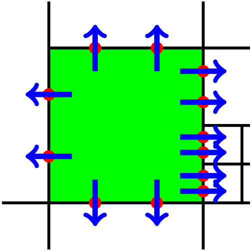
For refinement



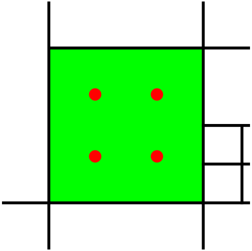
For wb quadrature in 1D

Reconstruction points

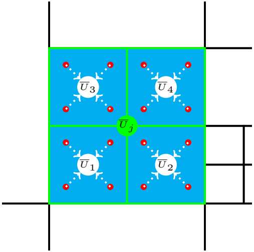
For fluxes



For source quadrature



For refinement



For wb quadrature in 1D

Reconstruction points

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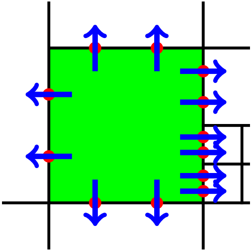
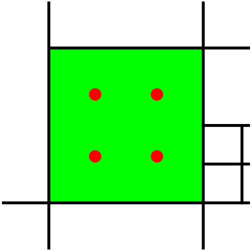


For source quadrature



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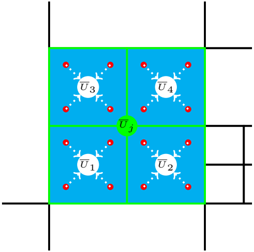
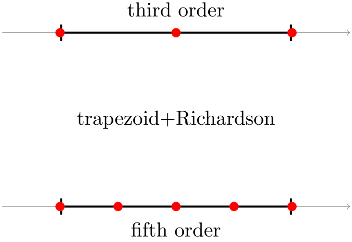
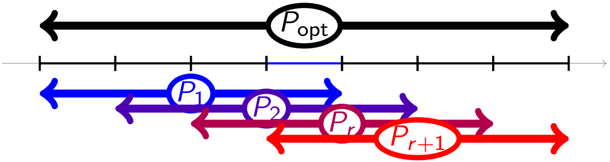


For wb quadrature in 1D



Essentially non-oscillatory reconstructions

Given the cell averages $\overline{u}_{j-r}, \dots, \overline{u}_{j+r}$ of a bounded function $u(x)$,



$$P_{\text{opt}} \text{ s.t. } \forall i = -r, \dots, r: \qquad \frac{1}{|\Omega_{j+i}|} \int_{\Omega_{j+i}} P_{\text{opt}}(x)\mathrm{d}x = \overline{u}_{j+i}$$

1. has accuracy $\mathcal{O}(\Delta x^{2r+1})$ in smooth regions
2. is however oscillatory if a discontinuity is present in its stencil
3. is best replaced by a (lower accuracy) non-oscillatory alternative, e.g. one of the P_k's

(Jiang-Shu) smoothness indicators

Given $P \in \mathbb{P}_N$,

Definition

$$\mathcal{I}[P] = \sum_{\ell=1}^{N} |\Omega_j|^{2\ell-1} \int_{\Omega_j} \left[\frac{\mathrm{d}^\ell}{\mathrm{d}x^\ell} P \right]^2 \mathrm{d}x$$

Properties

$$\mathcal{I}[P] = \mathcal{O}(1) \qquad \text{(in general)}$$
$$\mathcal{I}[P] \asymp 1 \qquad \text{(on discontinous data)}$$
$$\mathcal{I}[P] = u' \, |\Omega_j|^2 + \mathcal{O}(|\Omega_j|^4), \qquad \text{(on regular data)}$$

Jiang, Shu J. Comput. Phys. 1996

WENO: the linear coefficients

The WENO construction is based on the following fact:

$$\boxed{P_{\text{opt}}(x_{j+1/2}) = \sum_{k=1}^{r+1} d_k P_k(x_{j+1/2})}$$

For example:

- WENO3: $d_1 = 1/3, d_2 = 2/3$
- WENO5: $d_1 = 1/10, d_2 = 3/5, d_3 = 3/10$
- up to order 9: tabulated in the literature
- general formula: published in 2012

Note: the coefficients for $x_{j-1/2}$ are different!

Shu ICASE report (1997)

Arandiga et al. SINUM (2012)

WENO3: the linear coefficients on a nonuniform mesh

For $x^-_{j+1/2}$:

$$d_1 = \frac{h_{j+1}}{h_{j-1} + h_j + h_{j+1}}$$

$$d_2 = \frac{h_{j-1} + h_j}{h_{j-1} + h_j + h_{j+1}}$$

For $x^+_{j-1/2}$:

$$d_1 = \frac{h_j + h_{j+1}}{h_{j-1} + h_j + h_{j+1}}$$

$$d_2 = \frac{h_{j-1}}{h_{j-1} + h_j + h_{j+1}}$$

They must be recomputed after each mesh adaption!

WENO5: the linear coefficients on a nonuniform mesh

For $x^{+}_{0-1/2}$:

$$d_1 = \frac{h_1(h_1+h_2)}{(h_{-2}+\ldots+h_2)(h_{-2}+\ldots+h_1)} \in [0,1]$$

$$d_2 = \frac{(h_{-2}+h_{-1}+h_0)(h_1+h_2)(h_{-2}+2h_{-1}+2h_0+2h_1+h_2)}{(h_{-2}+\ldots+h_2)(h_{-1}+\ldots+h_2)(h_{-2}+\ldots+h_1)} \in [0,1]$$

$$d_3 = \frac{(h_{-2}+h_{-1}+h_0)(h_{-1}+h_0)}{(h_{-2}+\ldots+h_2)(h_{-1}+\ldots+h_2)} \in [0,1]$$

...and an alogous set for $x^{-}_{0+1/2}$...

WENO reconstructions

Given a point $\hat{x} \in \Omega_j$: The WENO reconstruction operator is

$$P_{\text{rec},j}(\hat{x}) = \text{WENO}(P_1, \ldots, P_{r+1}; P_{\text{opt}}, \hat{x}) \in \mathbb{R}$$

and is computed as follows:

1. Find optimal coefficients $d_1(\hat{x}), \ldots, d_{r+1}(\hat{x})$ such that
$$\sum_{k=1}^{r+1} d_k(\hat{x}) P_k(\hat{x}) = P_{\text{opt}}(\hat{x}) \text{ and } \sum_{k=1}^{r+1} d_k(\hat{x}) = 1.$$
2. Compute nonlinear coefficients ω_k as
$$\alpha_k(\hat{x}) = \frac{d_k(\hat{x})}{(I_{P_k} + \epsilon)^t} \qquad \omega_k(\hat{x}) = \frac{\alpha_k(\hat{x})}{\sum_{j=1}^{r+1} \alpha_j(\hat{x})}, \tag{1}$$
where $\mathcal{I}[P_k]$ denotes a suitable regularity indicator (later) evaluated on P_k, ϵ is a small positive quantity and $t \geq 2$.
3. Finally
$$P_{\text{rec},j}(\hat{x}) = \sum_{k=1}^{r+1} \omega_k(\hat{x}) P_k(\hat{x}) \tag{2}$$

WENO: smooth data

Goal: show that $u(\hat{x}) - P_{\text{rec}}(\hat{x}) = \mathcal{O}(h^{2r+1})$.

$$\overbrace{u(\hat{x}) - P_{\text{rec}}(\hat{x})}^{\substack{\text{reconstruction} \\ \text{error}}} = \underbrace{u(\hat{x}) - P_{\text{opt}}(\hat{x})}_{\mathcal{O}(h^{2r+1})} + P_{\text{opt}}(\hat{x}) - P_{\text{rec}}(\hat{x})$$

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$$= \mathcal{O}(h^{2r+1})) + \sum_k (d_k - \omega_k) P_k(\hat{x})$$

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$$= \mathcal{O}(h^{2r+1})) + \sum_k (d_k - \omega_k) P_k(\hat{x}) + \underbrace{\sum_k d_k}_{=1} u(\hat{x}) - \underbrace{\sum_k \omega_k}_{=1} u(\hat{x})$$

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$$= \mathcal{O}(h^{2r+1})) + \sum_k (d_k - \omega_k) P_k(\hat{x}) + \underbrace{\sum_k d_k}_{=1} u(\hat{x}) - \underbrace{\sum_k \omega_k}_{=1} u(\hat{x})$$

$$= \mathcal{O}(h^{2r+1}) + \sum_k \underbrace{(d_k - \omega_k)}_{\mathcal{O}(h^r)?} \underbrace{(P_k(\hat{x}) - u(\hat{x}))}_{\mathcal{O}(h^{r+1})}$$

WENO: smooth data (cont.)

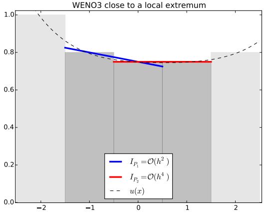
Arandiga et al: compare each I_k with some fixed $I_{\hat{k}}$

$$\omega_k = d_k \left[1 - d_k \frac{I_k - I_{\hat{k}}}{\epsilon + I_{\hat{k}}} \sum_{s=0}^{t-1} \left(\frac{\epsilon + I_k}{\epsilon + I_{\hat{k}}} \right)^s + \mathrm{o}\left(\frac{I_k - I_{\hat{k}}}{\epsilon + I_{\hat{k}}} \right) \right]$$

$$I_P = u'^2 h^2 + c_1 u' u'' h^3 + (c_2 u' u''' + c_3 u''^2) h^4 + \dots$$

Obstructions to $d_k - \omega_k = \mathcal{O}(h^r)$ come from

- differences between the indicators of two candidate polynomials
- ϵ too small



Arandiga et al. SINUM (2012)

WENO: discontinuous data

In this example:

$$I_{P_1} \asymp 1, \quad I_{P_2} \asymp 1, \quad I_{P_3} \asymp h^2$$

$$\alpha_1 \asymp 1, \quad \alpha_2 \asymp 1, \quad \alpha_3 \asymp {}^1\!/_{h^2}$$

$$\alpha_1 + \alpha_2 + \alpha_3 \asymp {}^1\!/_{h^2}$$

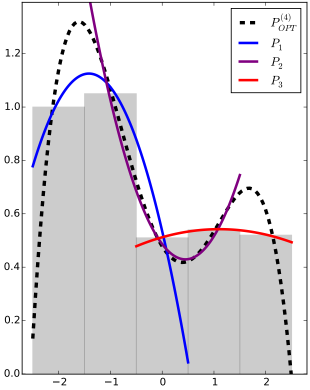
$$\omega_1 \asymp h^2, \quad \omega_2 \asymp h^2, \quad \omega_3 \asymp 1$$

$$\Downarrow$$

$$P_{\text{rec}} \simeq P_3$$

$$\Downarrow$$

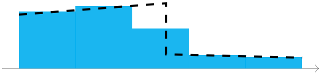
TVB reconstruction



Note!

It is crucial that at least one of $I_{P_1}, \ldots, I_{P_{r+1}}$ be $\mathcal{O}(h^2)$!

WENO: discontinuity in the central cell



- all candidate polynomials contain a discontinuity
- $\Rightarrow \quad \forall k : \mathcal{I}[P_k] \asymp 1$

- in the case of finite differences,
  all candidate polynomials are monotone in the cell[1]
  $\Rightarrow$ monotone reconstructed values

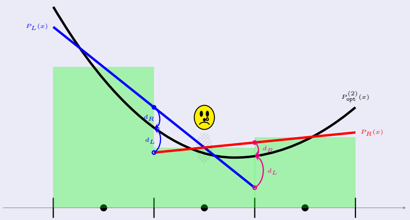
- in the case of finite volumes,
  very small over/undershoots can be created
  $\Rightarrow$ TVB reconstructed values

[1] Harten, Osher, Engquist, and Chachravarty. *Uniformly high order accurate essentially non-oscillatory schemes III*, NASA ICASE report 86-22 (1986).

Linear coefficients for cell center

WENO3

It is impossible to find d_L, d_R such that $P^{(2)}(x_j) = d_L P_L^{(1)}(x_j) + d_R P_R^{(1)}(x_j)$



WENO5

d_0, d_1, d_2 exist but are not in $[0, 1]$.
(See for a partial fix reducing the order to 4)

G. Puppo, M.S. Well-balanced high order 1D schemes on non-uniform grids and entropy residuals J Sci Comp (2016)

An extra candidate polynomial: P_0

Motivated from central schemes,

$$P_{\mathrm{opt}}(\hat{x}) = d_1(\hat{x})P_1(\hat{x}) + d_2(\hat{x})P_2(\hat{x}) \qquad \text{(WENO3)}$$

was replaced by

$$\forall x : P_{\mathrm{opt}}(x) = d_0 P_0(x) + d_1 P_1(x) + d_2 P_2(x) \qquad \text{(CWENO3)}$$

where

$$P_0(x) := \frac{1}{d_0}\left(P_{\mathrm{opt}}(x) - d_1 P_1(x) - d_2 P_2(x)\right)$$

Levy, Puppo, Russo M2AN (1999)

An extra candidate polynomial: P_0

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$$P_0(x) := \frac{1}{d_0} \left(P_{\text{opt}}(x) - d_1 P_1(x) - d_2 P_2(x) \right)$$

has only the following interpolation property:

$$\frac{1}{|\Omega_j|} \int_{\Omega_j} P_0(x) \mathrm{d}x = \overline{u}_j$$

Levy, Puppo, Russo M2AN (1999)

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Note that d_k do not depend on the reconstruction point,

Levy, Puppo, Russo M2AN (1999)

CWENO$(P_{\text{opt}}(x), P_1(x), \dots, P_n(x)) \longrightarrow P_{\text{rec}}(x)$

Given any $d_0, d_1, d_n \in (0,1)$ such that $\sum_{\xi=0}^{n} d_\xi = 1$:

1. $$P_0(x) := \frac{1}{d_0}\left(P_{\text{opt}}(x) - \sum_{\xi=1}^{n} d_\xi P_\xi(x)\right)$$
2. Compute smoothness indicators $I_0, I_1, \dots, I_n$
3. Compute nonlinear weights
$$\alpha_\xi = \frac{d_\xi}{(I_\xi+\epsilon)^t}, \qquad \omega_\xi = \frac{\alpha_\xi}{\sum_\lambda \alpha_\lambda}$$
4. Reconstruction polynomial (unif. accurate in the cell!)
$$\forall x \in \text{cell} \qquad P_{\text{rec}}(x) = \sum_{\xi=0}^{n} \omega_\xi P_\xi(x)$$

Levy, Puppo, Russo M2AN (1999)

G. Puppo, M.S. J. Sci. Comput. (2015, electronic)

Comparision of WENO and CWENO

WENO

- d_k depend on reconstruction point

CWENO

- d_k arbitrary (e.g. $d_0 = \frac{1}{2}, d_k = \frac{1}{2r}$)

Comparision of WENO and CWENO

WENO

- d_k depend on reconstruction point
- for each reconstruction point $\hat{x}$, need to compute $\omega_k(\hat{x})$ from $d_k(\hat{x})$

CWENO

- d_k arbitrary (e.g. $d_0 = \frac{1}{2}, d_k = \frac{1}{2r}$)
- compute ω_k from d_k only once per cell

Comparision of WENO and CWENO

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CWENO

- d_k arbitrary (e.g. $d_0 = \frac{1}{2}, d_k = \frac{1}{2r}$)
- compute ω_k from d_k only once per cell
- one extra regularity indicator to compute $\mathcal{I}[P_0]$

Comparision of WENO and CWENO

WENO

- d_k depend on reconstruction point
- for each reconstruction point $\hat{x}$, need to compute $\omega_k(\hat{x})$ from $d_k(\hat{x})$
- wonderful for conservation laws on structured cartesian meshes

CWENO

- d_k arbitrary (e.g. $d_0 = \frac{1}{2}, d_k = \frac{1}{2r}$)
- compute ω_k from d_k only once per cell
- one extra regularity indicator to compute $\mathcal{I}[P_0]$
- more suitable for balance laws, AMR, unstructured meshes, ...

1D CWENO in the literature

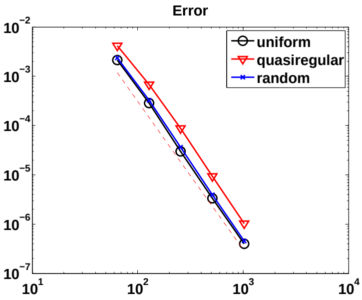
- Levy, Puppo, Russo order 3, uniform mesh M2AN (1999)
- Capdeville order 5, non uniform mesh JCP (2008)
- Zaharan WENO-Z nonlinear weights Appl. Math. Comp. (2009)
- Kolb Analysis of CWENO3 on uniform meshes (choice of ϵ) SINUM (2015)
- I. Cravero, M.S. Analysis of CWENO3 on non-uniform meshes (choice of ϵ) J Sci Comput (2016)
- I. Cravero, G. Puppo, M.S., G. Visconti CWENO5, CWENO7, Hierarchic CWENO, ... In preparation

Multi-dimensional CWENO in the literature

- Levy, Puppo, Russo order 3, 2D, cartesian mesh SIAM J Sci Comput (2000)
- Levy, Puppo, Russo order 4, 2D, cartesian mesh SIAM J Sci Comput (2002)
- Lahooti, Pishevar order 4, 3D, cartesian mesh Appl. Math. and Comput (2012)
- M.S., Coco, Russo order 3, 2D, quad-tree mesh J Sci Comput (2016)
- M. Dumbser, M.S. CWENO3 on triangular and tetrahedral meshes In preparation

Shallow water equations using CWENO3

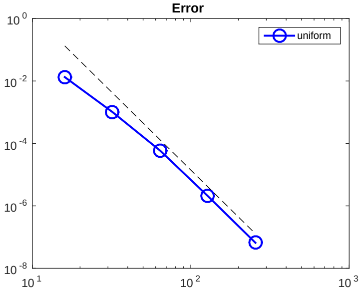
Exploiting the
reconstruction at cell center,
we can build a
well-balanced scheme
with third order accuracy:



	$\|\Delta(h+z)\|_\infty$				$\|q\|_\infty$			
Q-regular	100	200	400	800	100	200	400	800
$p=3$	0	4.44e-16	4.44e-16	6.66e-16	6.87e-16	1.47e-15	1.67e-15	2.47e-15
Random								
$p=3$	2.22e-16	6.66e-16	6.66e-16	6.66e-16	5.63e-16	8.47e-16	9.94e-16	1.28e-15

Shallow water equations using CWENO5

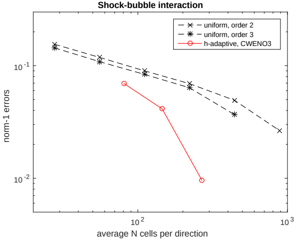
Exploiting the reconstruction at $x_{j-1/4}, x_j, x_{j+1/4}$, we can build a well-balanced scheme with fifth order accuracy:

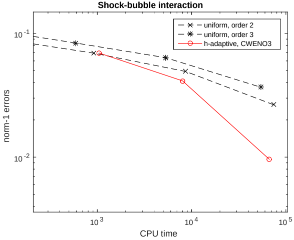


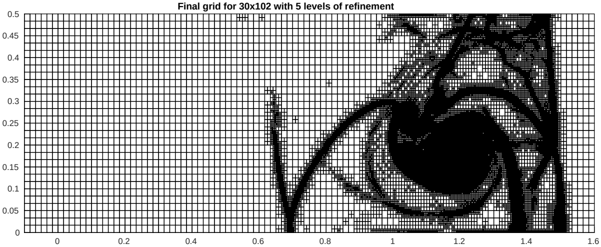
	$\|\Delta(h+z)\|_\infty$				$\|q\|_\infty$			
	100	200	400	800	100	200	400	800
$p=5$	2.22e-16	8.88e-16	1.33e-15	1.11e-16	6.10e-16	8.21e-16	9.69e-16	1.24e-15

2D Euler equations with AMR on quad-tree grids

Shock-bubble interaction test for 2D Euler equations







M.S., Coco, Russo, order 3, 2D, quad-tree mesh J Sci Comput (2016)

CWENO on triangular unstructured meshes

Thanks to M. Dumbser!

CWENO on triangular unstructured meshes

Thanks to M. Dumbser!

	$\mathbb{P}_2$		$\mathbb{P}_3$	
	9.78E-02		4.95E-02	
	1.96E-02	2.33	1.73E-03	4.03
	2.88E-03	2.80	1.13E-04	3.86
	3.71E-04	2.97	6.82E-06	4.11

	$\mathbb{P}_4$		$\mathbb{P}_5$	
	6.12E-02		5.87E-02	
	1.55E-03	4.92	9.25E-04	6.56
	6.13E-05	4.77	5.46E-06	6.96
	1.93E-06	5.03	FR 4703	

CWENO on discontinuous data (jump not in central cell)

WENO argument

For $k = 1, \ldots, r$:

- at least one of P_k's iterpolates smooth data, so $\mathcal{I}[P_k]\,\mathcal{O}(h^2)$
- all P_k's interpolationg across the jump will have $\mathcal{I}[P_k] \asymp 1$
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CWENO argument?

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- reconstruction (essentially uses) only information from the smooth part, only if also $\mathcal{I}[P_0] \asymp 1$!

Property A

Property A

If $\mathcal{I}[P_{\text{opt}}] \asymp 1$, then $\mathcal{I}[P_0] \asymp 1$

- sufficient to prove "ENO" property of CWENO
- it looks trivial, but

$$P_0 := \frac{1}{d_0}\left(P_{\text{opt}} - \sum_{k=1}^{r} d_k P_k\right)$$

$$\mathcal{I}[P_0] := \sum_{\ell=1}^{N} h^{2\ell-1} \int_{\Omega_j} \left[\frac{\mathrm{d}^\ell}{\mathrm{d}x^\ell} P_0\right]^2 \mathrm{d}x$$

$$= \frac{1}{d_0}\mathcal{I}[P_{\text{opt}}] + \sum_{k=1}^{r} \frac{d_k}{d_0}\mathcal{I}[P_{\text{opt}}] + \text{cross terms}$$

- and the cross terms do not have a definite sign

Proofs of Property A

Proposition (CWENO3)

For any choice of weights, explicit computation yields

$$\frac{\mathcal{I}[P_0]}{\mathcal{I}[P_{\text{opt}}]} = \frac{3d_0^2 - 6d_0 + 16}{16d_0^2} > \frac{13}{16}$$

Proposition (CWENO5)

For any (symmetric) choice of weights, then $\dfrac{\mathcal{I}[P_0]}{\mathcal{I}[P_{\text{opt}}]}$ attains its minimum for $d_0 = 1$ and

$$\left.\frac{\mathcal{I}[P_0]}{\mathcal{I}[P_{\text{opt}}]}\right|_{d_0=1} > 0.6.$$

Proposition (CWENO7)

A similar result holds true...

I.C., G.P., M.S., G.V. In preparation

Conclusions and perspectives

- CWENO is much more flexible than WENO and can be applied in more general situations
- Accuracy on smooth data can be proven similarly as for WENO and depends on ϵ (best: $\epsilon = h^2$)
- Non-oscillatory properties depend on a new idea *Property A* that we could prove for each "reasonable" 1D CWENO's
- CWENO of aerbitrary order on triangular and tetrahedral meshes has been implemented this week
- A new hierarchic construction CWENOH tailored to data with smooth high frequencies and discontinuities will come soon. . .

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Thanks for the attention!