A non-conservative flux approach ensuring the C-property for the lake at rest in the framework of a MOOD based 2D shallow-water 6th-order FV scheme

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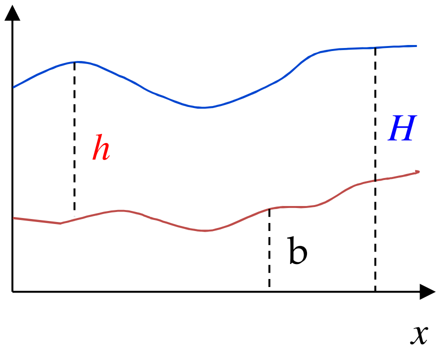
- Presentation Outline

- The shallow-water problem
- Finite volume scheme
- The MOOD procedure
- Numerical simulations
- Conclusions

- The shallow-water problem (1)

$$\begin{cases} \partial_t h + \partial_x (hu) + \partial_y (hv) = 0 \\ \partial_t (hu) + \partial_x \left(hu^2 + \frac{g}{2} h^2 \right) + \partial_y (huv) = -gh\partial_x b \\ \partial_t (hv) + \partial_x (huv) + \partial_y \left(hv^2 + \frac{g}{2} h^2 \right) = -gh\partial_y b \end{cases}$$



$$\mathrm{H}(x,y,t) = h(x,y,t) + b(x,y)$$

We **rewrite the problem** as:

$$\begin{cases} \partial_t H + \partial_x (hu) + \partial_y (hv) = 0 \\ \partial_t (hu) + \partial_x \left(hu^2 + \frac{gH}{2} (H - 2b) \right) + \partial_y (huv) = -gH\partial_x b \\ \partial_t (hv) + \partial_x (huv) + \partial_y \left(hv^2 + \frac{gH}{2} (H - 2b) \right) = -gH\partial_y b \end{cases}$$

- The shallow-water problem (2)

Defining $V=\left(H,\ hu,\ hv,\ b\right)^T$, $\mathrm{U}=\left(u,\ v\right)^T$, and

$$F=\begin{pmatrix} hu \\ hu^2+\frac{gH}{2}(H-2b) \\ huv \\ 0 \end{pmatrix},\quad \mathrm{G}=\begin{pmatrix} hv \\ huv \\ hv^2+\frac{gH}{2}(H-2b) \\ 0 \end{pmatrix},\quad S=-gH\begin{pmatrix} 0 \\ \partial_x \mathrm{b} \\ \partial_y \mathrm{b} \\ 0 \end{pmatrix}$$

leads to the compact form:

$$\partial_t V+\partial_x F(V)+\partial_y G(V)=S(V)$$

For "lake at rest" conditions: $H(x,y,t)=const,\quad U(x,y,t)=0:$

$$\partial_t V=-\partial_x F(V)-\partial_y G(V)+S(V)=0$$

- Finite volume scheme (1)

Mesh notations:

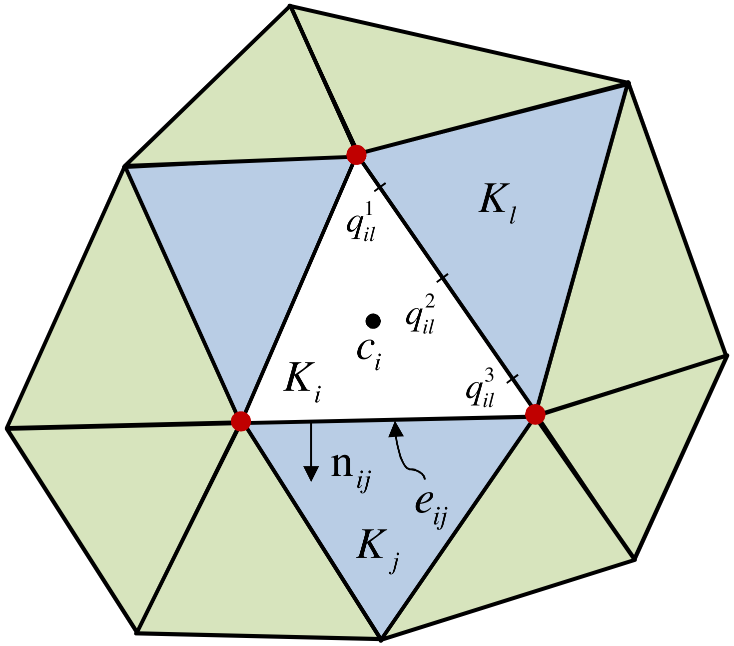
- Mesh τ of triangle polyhedral cells: K_i, area: $|K_i|$, centroid: c_i

$e_{ij} = K_i \cap K_j$

n_{ij}

q_{ij}^r

$\nu_e(i)$



- Finite volume scheme (2)

FV scheme:

$$\mathbf{F}\left(V;n\right) = F(V)\cdot n_x + G(V)\cdot n_y$$

$$\rightarrow \ \partial_t V = -\nabla\cdot\mathbf{F} + S \ \rightarrow \ \int_{K_i}\partial_t V = -\sum_{j\in\nu_e(i)}\int_{e_{ij}}\mathbf{F}\cdot n_{ij} + \int_{K_i} S$$

Defining **cell/edge averages**:

$$V_i^t = \frac{1}{|K_i|}\int_{K_i} V, \quad F_{ij}^t = \frac{1}{|e_{ij}|}\int_{e_{ij}}\mathbf{F}\cdot n_{ij}, \quad S_i^t = \frac{1}{|K_i|}\int_{K_i} S$$

$$\rightarrow \ \boxed{\frac{dV_i^t}{dt} = -\sum_{j\in\nu_e(i)}\frac{|e_{ij}|}{|K_i|}F_{ij}^t + S_i^t}$$

- Finite volume scheme (3)

FV scheme:

Time/space **discretization**:

$$V_i^n \approx V_i^{t^n}$$

$$\mathcal{F}_{ij}^n(V_i^n, V_j^n; n_{ij}) \approx F_{ij}^{t^n} \qquad \rightarrow \text{ conservative; depends on flux model}$$

$$S_i^n(V_i^n) \approx S_i^{t^n} \qquad \rightarrow S_i^n = -g\int_{K_i} H_i^n \left(0,\ \partial_x b_i,\ \partial_y b_i,\ 0\right)^T$$

leads to

$$\rightarrow \quad \boxed{V_i^{n+1} = V_i^n - \Delta t \left(\sum_{j \in \nu_e(i)} \frac{|e_{ij}|}{|K_i|} \sum_{r=1}^{N_{GP}} w_r \mathcal{F}_{ij,r}^n \right) + \Delta t\, S_i^n} \qquad \text{(RK1)}$$

- Finite volume scheme (4)

The Polynomial Reconstruction Operator (PRO)

For the evaluation of edge and cell integrals we use **quadrature formulas** that require estimated values of

$$H,\ q_x \equiv hu,\ q_y \equiv hv,\ b,\ h$$

at the corresponding **Gauss points**.

For each cell we use a **conservative polynomial reconstruction** of the variables, of **degree up to 5**, based on the **average values of the cells** surrounding the cell.

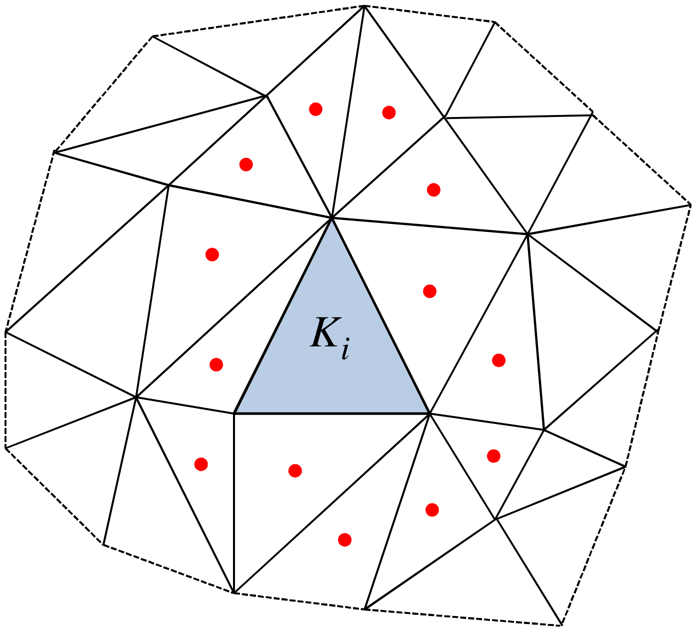
This reconstruction is done: $H, q_x, q_y, b \ \rightarrow\ \hat{H}, \hat{q}_x, \hat{q}_y, \hat{b}$

To preserve the **hydrostatic condition** (needed for "**C-property**"): $\hat{h} = \hat{H} - \hat{b}$

- Finite volume scheme (5)

The Polynomial Reconstruction Operator (PRO)

Stencil and minimization scheme (LSM) for computation of polynomial coefficients for a given cell:



- Finite volume scheme (6)

For "lake at rest" conditions, in general, we get from the FV scheme:

$$\rightarrow \quad \frac{V_i^{n+1} - V_i^n}{\Delta t} = S_i^n - \sum_{j \in \nu_e(i)} \frac{|e_{ij}|}{|K_i|} \sum_{r=1}^{N_{GP}} w_r \mathcal{F}_{ij,r}^n \neq 0$$

Although, for smooth bathymetry, the scheme shows the expected order of accuracy ("well-balanced").

How to solve?

- Finite volume scheme (7)

- Need to **correct** somehow the **source term discretization**

↓

- Introduce a **numerical non-conservative flux** $\mathcal{E}$

↓

- Strong dependence on the numerical flux used to compute $\mathcal{F}$

↓

- $\mathcal{F}$ has to fulfill a **new property**

Physical bathymetry representative (PBR):

(assume lake at rest)

$$V_L = (H, 0, 0, b_L) \;\Big|\; V_R = (H, 0, 0, b_R)$$

$\mathcal{F}$ admits a PBR if there exists $b^* \in \mathbb{R}$, such that the state

$V^* = (H, 0, 0, b^*)$ satisfies

$$\boxed{\mathcal{F}(V_L, V_R; n) = F(V^*) \cdot n_x + G(V^*) \cdot n_y}$$

- Finite volume scheme (8)

- It can be shown this **property holds for the popular numerical fluxes**, namely, Rusanov, HLL, HLL+C, and HLLC. Moreover, for these fluxes,

$$b^* = (1-\theta)b_L + \theta b_R, \quad \theta \in [0,1]$$

- **Main result**: defining, for each Gauss point of each edge e_{ij}

$$\mathcal{E}_{ij,r}^n = -gH_{ij,r}^n \left(0,\ \left(b_{ij,r}^* - b_{ij,r}\right) n_{ij},\ 0\right)^T$$

one obtains that the following scheme satisfies the "C-property":

$$\rightarrow \quad V_i^{n+1} = V_i^n - \Delta t \left(\sum_{j \in \nu_e(i)} \frac{|e_{ij}|}{|K_i|} \sum_{r=1}^{N_{GP}} w_r \left(\mathcal{F}_{ij,r}^n + \mathcal{E}_{ij,r}^n \right) \right) + \Delta t\, S_i^n$$

- The (PRO-) MOOD procedure (1)

Why use **MOOD (Multi-dimensional Optimal Order Detection)**?

Numerical solutions for the **SW problem often present discontinuities**

↳ Very high-order spatial schemes do not make sense in the vicinity of **discontinuities**

↳ One has to recover (in the limit case) the first-order FV scheme to have **stability**

MOOD purpose:

- **detection** of cells where high-order polynomial reconstruction is not eligible

MOOD main feature (Clain, Diot & Loubère 2011):

- ***a posteriori* detection procedure** for the eligibility of the solution

- The (PRO-) MOOD procedure (2)

The MOOD procedure in brief:

A solution is eligible if:

1: is physically admissible (**PAD**) - mandatory

2: one has a local discrete maximum principle (**DMP**) for the solution

3: if (2) fails, the **solution is still admissible** if it is locally considered as:

- non-oscillating
- C^2

} → **u2 detection procedure**

- The (PRO-) MOOD procedure (3)

MOOD procedure in detail (DMP): starting point $\longrightarrow$ numerical solution $W^n = (W_i^n)$

Step 1: mark each cell K_i with the polynomial degree $d_i = d_{\max}$ for PRO

Step 2: compute polynomial reconstruction of degree d_i for W_i at $t = t^n$

Step 3: evaluate the flux approximation at the Gauss points and the source term

Step 4: compute a **candidate solution** $W^* = (W_i^*)$ at $t = t^{n+1}$

Step 5: the solution $H^*_i, (hu)^*_i, (hv)^*_i$ is considered **eligible** if:

- the solution is physically admissible (PAD): $h_i^* \geq 0$
- we have a local maximum principle (DMP) for h_i^*: $\min\limits_{j \in \nu(i)} \left(h_i^n, h_j^n\right) \leq h_i^* \leq \max\limits_{j \in \nu(i)} \left(h_i^n, h_j^n\right)$

Step 6: if the solution is not eligible: $d_i \rightarrow d_i - 1$. For $d_i = 0$ PAD and DMP are both satisfied since we employ **positive preserving fluxes** that respect DMP for the eight

- The (PRO-) MOOD procedure (4)

MOOD procedure in detail (u2 detection process):

DMP, although needed when the solution in not smooth, **can lead to a low-order scheme** because it **does not distinguish** between **smooth/non-smooth extrema**.

⬇

u2 detection process (Diot, Clain & Loubère 2012):

i) W_i^* not respecting PAD implies polynomial degree is decremented in cell

ii) W_i^* respecting PAD but not DMP is nevertheless **considered eligible** if:

- is locally considered as: non-oscillating (C1) and C^2 (C2)

Diot et al. proposed numerical counterparts for C1 and C2 that intend to overcome the difficulty that arises from the fact that a function may be considered irregular for coarse mesh but regular for thinner one. The numerical criteria involve the "curvatures" of the $\mathbb{P}_2$ reconstruction of function W, denoted $\tilde{W}$, *i.e.* $\partial_{xx}\tilde{W}$ and $\partial_{yy}\tilde{W}$, for cells "around" cell i.

- NS: C-property / Lake at rest (1)

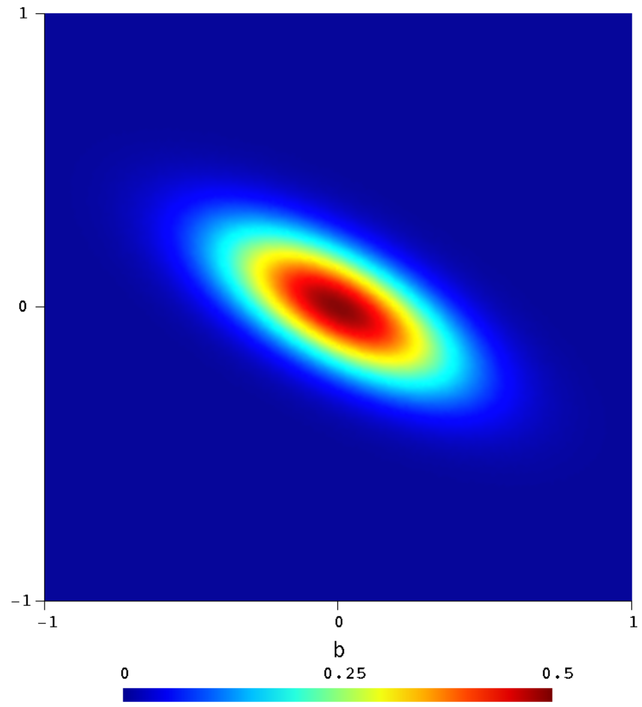
Purpose: check that the steady-state solution $(V=0)$ is strictly preserved for an initial flat free surface - **non flat bottom**.

Problem geometry and bathymetry:

$$b(x,y)=0.5e^{-3(4x^2+8xy+9y^2)}$$

↓

smooth case



Initial conditions: $H=1,\ U=0$

Boundary: reflection conditions

Other: tested with various reconstruction degrees and flux schemes

- NS: C-property / Lake at rest (2)

Error definition for convergence analysis:

$$err_1 = \frac{\sum_i \left|\varphi^t_{num,i} - \varphi^t_{exact,i}\right| \, \left|K_i\right|}{\sum_i \left|K_i\right|}, \qquad err_\infty = \max_i \left|\varphi^t_{num,i} - \varphi^t_{exact,i}\right|$$

Performed simulation for **15 seconds** for two meshes (2500 and 10050 triangles) and different polynomial reconstruction degrees for unknowns.

Using the "corrected" scheme, we found (using Rusanov, HLL, HLLC fluxes)

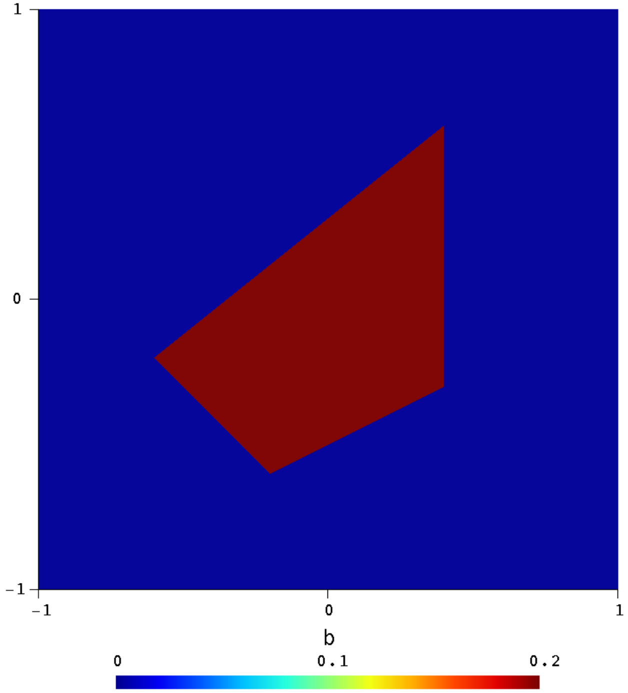
$$err_\infty(h) < 10^{-12}, \;\; err_\infty(hu) < 10^{-11}, \;\; err_\infty(hv) < 10^{-11}$$

Using the "standard" scheme, we obtained (maximum errors):

$$err_\infty(h) \approx 0.002, \;\; err_\infty(u) \approx 0.07, \;\; err_\infty(v) \approx 0.07$$

- NS: C-property / Lake at rest (3)

Problem geometry and bathymetry:



non-smooth test cases:

$$(i)\ b_{\max} = 0.2,\quad (ii)\ b_{\max} = 0.5$$

Initial conditions: $H = 1,\ U = 0$

- NS: C-property / Lake at rest (4)

Performed simulation for **15 seconds** for two meshes (2594 and 10352 triangles) and different polynomial reconstruction degrees for unknowns.

Case (i):

Using the "corrected" scheme:

$$err_\infty(h) < 10^{-12}, \;\; err_\infty(hu) < 10^{-11}, \;\; err_\infty(hv) < 10^{-11}$$

Using the "standard" scheme (maximum errors):

$$err_\infty(h) \approx 0.3, \;\; err_\infty(u) \approx 1.5, \;\; err_\infty(v) \approx 1.5$$

Case (ii): similar results to Case (i) for "**corrected**" scheme, and, in general, **no convergence** for "**standard**" scheme

- NS: C-property / Convergence rate (1)

Purpose: check convergence rate of the "corrected" scheme for a smooth bathymetry when the **water level does not depend on coordinates**, but **velocity does** (Castro, Toro & Käser 2012)

Problem geometry: $(x,y)\in[-1,1]\times[-1,1],\ \ b=e^{-8(x^2+y^2)}$

Initial conditions/solution: **Source term set** such that

$$H=e^{0.1t},\ \ u=0.2+0.1\sin(\pi x),\ \ v=0.2+0.1\sin(\pi y)$$

Boundary: Dirichlet boundary conditions (imposed on edge Gauss points)

Other: $\mathbb{P}_5$ for bathymetry, simulation up to 0.5 seconds, HLLC flux.

- NS: C-property / Convergence rate (2)

Convergence results

P2 – h				
cells	err_∞	order	err_2	order
5202	1.25E-5	-	3.63E-6	-
9248	5.31E-6	**3.0**	1.55E-6	**3.0**
14450	2.72E-6	**3.0**	7.95E-7	**3.0**
20808	1.57E-6	**3.0**	4.61E-7	**3.0**

P2 – hu				
cells	err_∞	order	err_2	order
5202	1.22E-4	-	2.86E-5	-
9248	5.26E-5	**2.9**	1.23E-5	**2.9**
14450	2.72E-5	**3.0**	6.38E-6	**2.9**
20808	1.58E-5	**3.0**	3.72E-6	**3.0**

P3 – h				
cells	err_∞	order	err_2	order
5202	3.86E-7	-	9.81E-8	-
9248	1.23E-7	**4.0**	3.15E-8	**4.0**
14450	4.92E-8	**4.1**	1.28E-8	**4.0**
20808	2.31E-8	**4.2**	6.12E-9	**4.1**

P3 – hu				
cells	err_∞	order	err_2	order
5202	1.13E-5	-	1.59E-6	-
9248	4.13E-6	**3.5**	5.38E-7	**3.8**
14450	1.74E-6	**3.9**	2.26E-7	**3.9**
20808	8.19E-7	**4.1**	1.10E-7	**4.0**

- NS: C-property / Convergence rate (3)

Convergence results

P4 – h				
cells	err_∞	order	err_2	order
5202	2.53E-7	-	3.55E-8	-
9248	6.34E-8	**4.8**	8.57E-9	**4.9**
14450	2.18E-8	**4.9**	2.83E-9	**5.0**
20808	8.65E-9	**4.9**	1.15E-9	**4.9**

P4 – hu				
cells	err_∞	order	err_2	order
5202	3.22E-6	-	5.39E-7	-
9248	8.65E-7	**4.6**	1.34E-7	**4.9**
14450	2.95E-7	**4.8**	4.41E-8	**5.0**
20808	1.21E-7	**4.9**	1.78E-8	**5.0**

P5 – h				
cells	err_∞	order	err_2	order
5202	7.90E-9	-	1.77E-9	-
9248	1.23E-9	**6.5**	2.91E-10	**6.3**
14450	3.19E-10	**6.1**	7.28E-11	**6.2**
20808	1.10E-10	**5.8**	2.39E-11	**6.1**

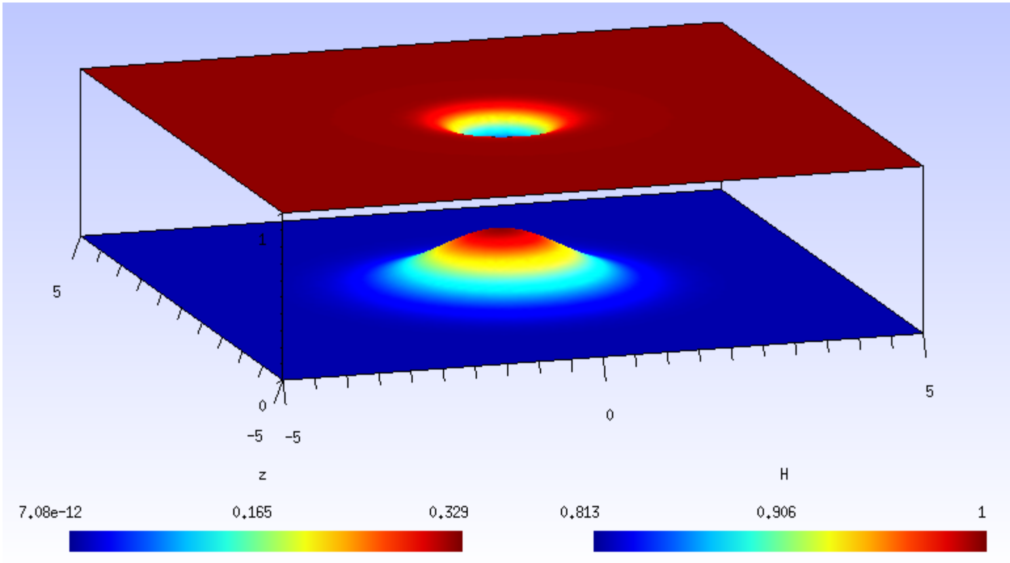
P5 – hu				
cells	err_∞	order	err_2	order
5202	1.24E-7	-	2.21E-8	-
9248	2.26E-8	**5.9**	3.62E-9	**6.3**
14450	5.52E-9	**6.3**	8.97E-10	**6.3**
20808	1.77E-9	**6.2**	2.92E-10	**6.2**

- NS: Stationary vortex (1)

Purpose: check convergence of scheme is stationary scenario - **non flat bottom**:

Problem geometry:

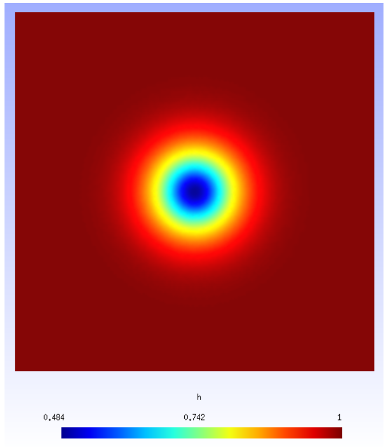
$$b = \frac{1}{5} e^{(1-x^2-y^2)/2}$$

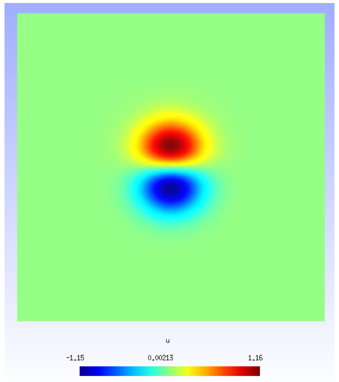


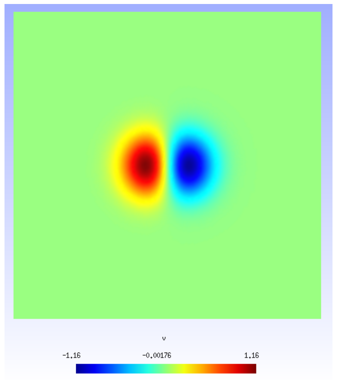
Initial conditions/solution:

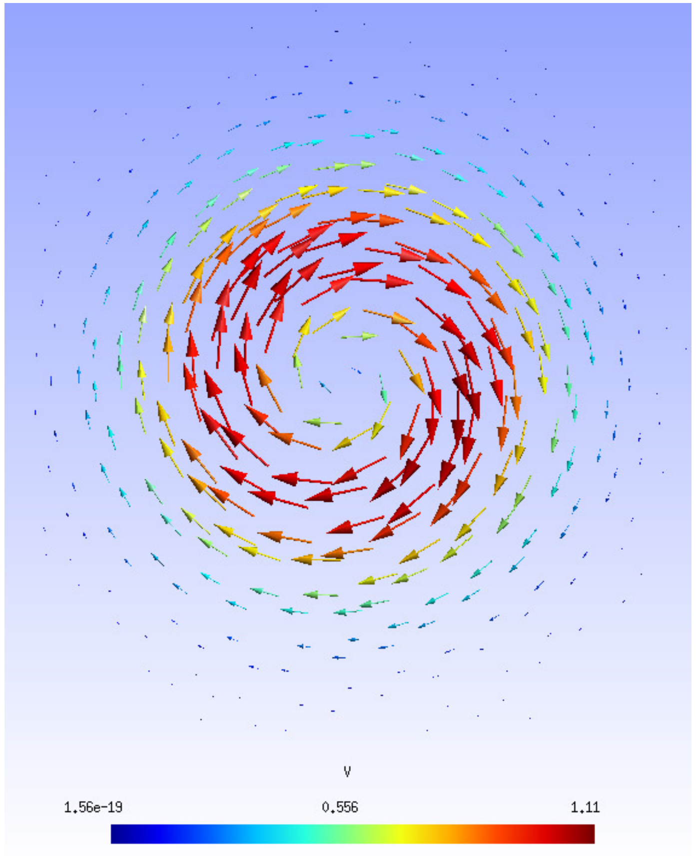
$$H = 1 - \frac{1}{4g} e^{2(1-x^2-y^2)}, \quad u = y e^{(1-x^2-y^2)}, \quad v = -x e^{(1-x^2-y^2)}$$

- NS: Stationary vortex (2)









- NS: Stationary vortex (3)

Convergence results $(t = 0.5)$ **:**

P2 – h				
cells	err_∞	order	err_2	order
524	8.29E-2	-	1.57E-3	-
2076	2.33E-2	**1.8**	3.98E-4	**2.0**
8238	2.74E-3	**3.1**	5.99E-5	**2.8**
33550	3.60E-4	**2.9**	7.56E-6	**3.0**

P3 – h				
cells	err_∞	order	err_2	order
524	3.77E-2	-	8.48E-4	-
2076	2.12E-3	**4.2**	5.10E-5	**4.1**
8238	1.55E-4	**3.8**	2.94E-6	**4.1**
33550	7.43E-6	**4.3**	1.55E-7	**4.2**

P4 – h				
cells	err_∞	order	err_2	order
524	3.90E-2	-	7.14E-4	-
2076	1.87E-3	**4.4**	4.20E-5	**4.1**
8238	1.25E-4	**4.0**	1.82E-6	**4.6**
33550	6.08E-6	**4.3**	6.11E-8	**4.8**

P5 – h				
cells	err_∞	order	err_2	order
524	1.98E-2	-	4.65E-4	-
2076	1.09E-3	**4.2**	1.40E-5	**5.1**
8238	1.09E-5	**6.7**	2.08E-7	**6.1**
33550	2.14E-7	**5.6**	3.24E-9	**5.9**

- NS: Moving vortex (1)

Purpose: check convergence in transient scenario - **flat bottom** (**u2** vs. **DMP**)

Problem geometry: same as static vortex but with flat bathymetry

Initial: $h = h_\infty - \frac{1}{4g} e^{2(1-x^2-y^2)}, \quad u = ye^{(1-x^2-y^2)} + u_\infty, \quad v = -xe^{(1-x^2-y^2)} + v_\infty$

where $h_\infty,\ u_\infty,\ v_\infty$ are constants

Sol.: $h = h_\infty - \frac{1}{4g} e^{2(1-r^2)}, \quad u = (y - v_\infty t)e^{(1-r^2)} + u_\infty, \quad v = -(x - u_\infty t)e^{(1-r^2)} + v_\infty$

with $r^2 = (x - u_\infty t)^2 + (y - v_\infty t)^2$

$h_\infty = 1, \quad u_\infty = 1, \quad v_\infty = 2, \quad t_{final} = 1$

- NS: Moving vortex (2)

Convergence results $(t = 1.0)$ **:**

P4 – h	**(u2)**			
cells	err_∞	order	err_2	order
524	8.86E-2	-	1.91E-3	-
2076	2.29E-2	**2.0**	1.19E-5	**4.0**
8238	1.76E-4	**7.0**	3.96E-6	**4.9**
33550	6.27E-6	**4.8**	1.35E-8	**4.8**

P4 – h	**(DMP)**			
cells	err_∞	order	err_2	order
524	8.94E-2	-	1.90E-3	-
2076	4.85E-2	**0.9**	4.08E-4	**2.2**
8238	1.09E-2	**2.2**	6.59E-5	**2.7**
33550	3.21E-3	**1.7**	1.52E-5	**2.1**

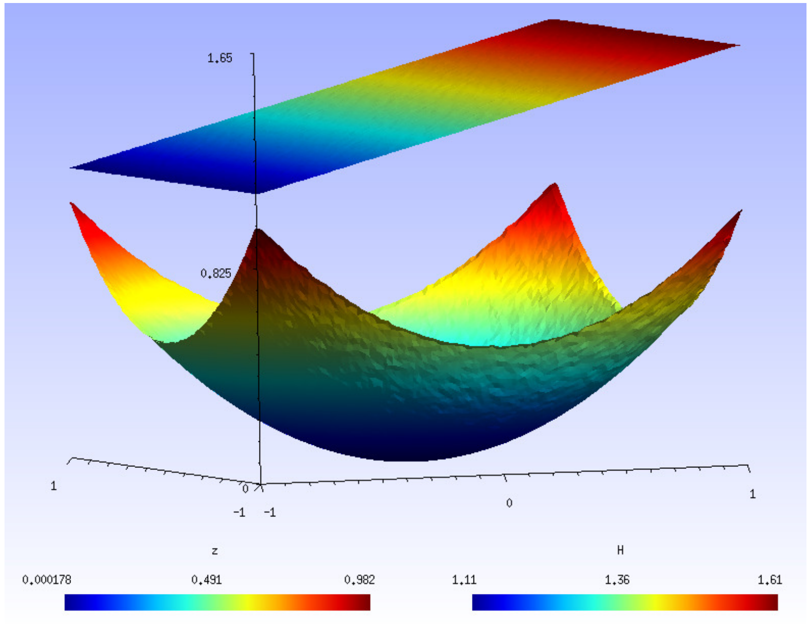
P5 – h	**(u2)**			
cells	err_∞	order	err_2	order
524	7.77E-2	-	1.67E-3	-
2076	2.51E-2	**1.6**	8.24E-5	**4.4**
8238	3.41E-5	**9.6**	6.55E-7	**7.0**
33550	4.38E-7	**6.2**	9.09E-9	**6.1**

P5 – h	**(DMP)**			
cells	err_∞	order	err_2	order
524	7.81E-2	-	1.58E-3	-
2076	4.58E-2	**0.8**	4.04E-4	**2.0**
8238	1.13E-2	**2.0**	8.40E-5	**2.3**
33550	3.94E-3	**1.5**	1.89E-5	**2.1**

- NS: Periodic solution (1)

Purpose: check convergence in transient scenario - **non flat bottom** (**u2** vs **DMP**)

Problem geometry: $b = \dfrac{w^2}{2g}(x^2 + y^2)$



Initial conditions/solution:

$$H = \frac{w^2}{g}\Big(A\big(x\cos(wt) + y\sin(wt)\big) + \alpha\Big)$$

$$u = -Aw\sin(wt), \quad v = Aw\cos(wt)$$

with

$$A = 0.25, \quad \alpha = 1.4, \quad w = 2\pi$$

final simulation time: $t_{final} = 2.125$

Other: $\mathbb{P}_2$ for b reconstruction

- NS: Periodic solution (2)

Convergence results $(t = 2.125)$:

P1 – h		**(PAD)**		
cells	err_∞	order	err_2	order
1040	1.19E-3	-	2.35E-4	-
4054	1.50E-3	-	3.15E-5	**3.0**
16264	9.38E-4	**0.7**	1.51E-5	**2.0**
65422	1.83E-4	**2.4**	3.70E-6	**2.0**

P1 – h		**(DMP)**		
cells	err_∞	order	err_2	order
1040	8.20E-3	-	3.09E-4	-
4054	4.72E-3	**0.8**	9.28E-5	**1.8**
16264	3.52E-3	**0.4**	2.68E-5	**1.8**
65422	2.19E-3	**0.7**	6.62E-6	**2.0**

P2 – h		**(u2)**		
cells	err_∞	order	err_2	order
1040	1.19E-6	-	6.92E-8	-
4054	1.19E-7	**3.4**	3.72E-9	**4.3**
16264	2.07E-8	**2.5**	3.63E-10	**3.4**

P2 – h		**(DMP)**		
cells	err_∞	order	err_2	order
1040	9.43E-3	-	2.64E-4	-
4054	6.92E-3	**0.5**	8.96E-5	**1.6**
16264	3.52E-3	**1.0**	2.50E-5	**1.8**

- NS: Periodic solution (3)

Convergence results – time scheme $(t = 2.125,\ 1040 \text{ cells, u2})$:

P2 + RK1 – h				
Δt	err_∞	order	err_2	order
CFL	1.93E-3	-	9.02E-4	-
CFL/2	9.61E-4	**1.0**	4.50E-4	**1.0**
CFL/4	4.80E-4	**1.0**	2.25E-4	**1.0**
CFL/8	2.40E-4	**1.0**	1.12E-4	**1.0**

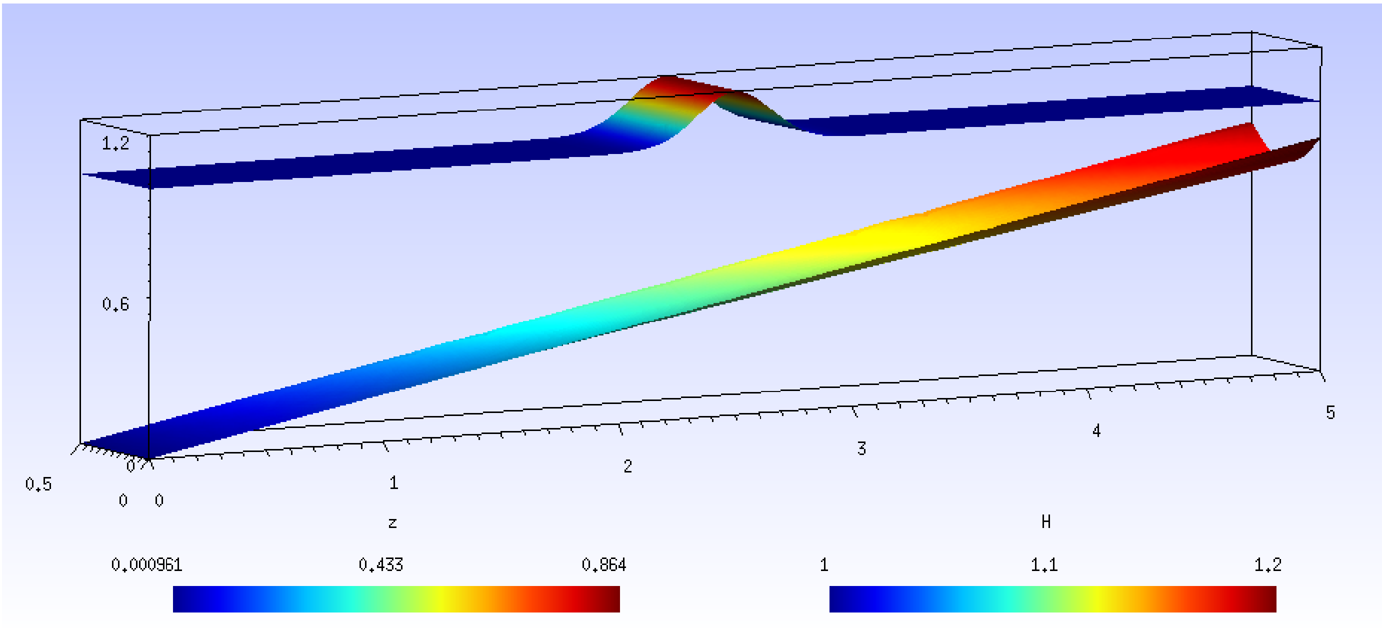
P4 + TVD-RK2 – h				
Δt	err_∞	order	err_2	order
CFL	1.25E-5	-	6.28E-6	-
CFL/2	2.87E-6	**2.1**	1.56E-6	**2.0**
CFL/4	6.92E-7	**2.1**	3.91E-7	**2.0**
CFL/8	1.71E-7	**2.0**	9.76E-8	**2.0**

P5 + TVD-RK3 – h				
Δt	err_∞	order	err_2	order
CFL	1.21E-6	-	5.65E-8	-
CFL/2	1.41E-7	**3.1**	6.85E-9	**3.0**
CFL/4	1.70E-8	**3.1**	8.49E-10	**3.0**
CFL/8	2.10E-9	**3.0**	1.06E-10	**3.0**

- NS: Travelling wave (1)

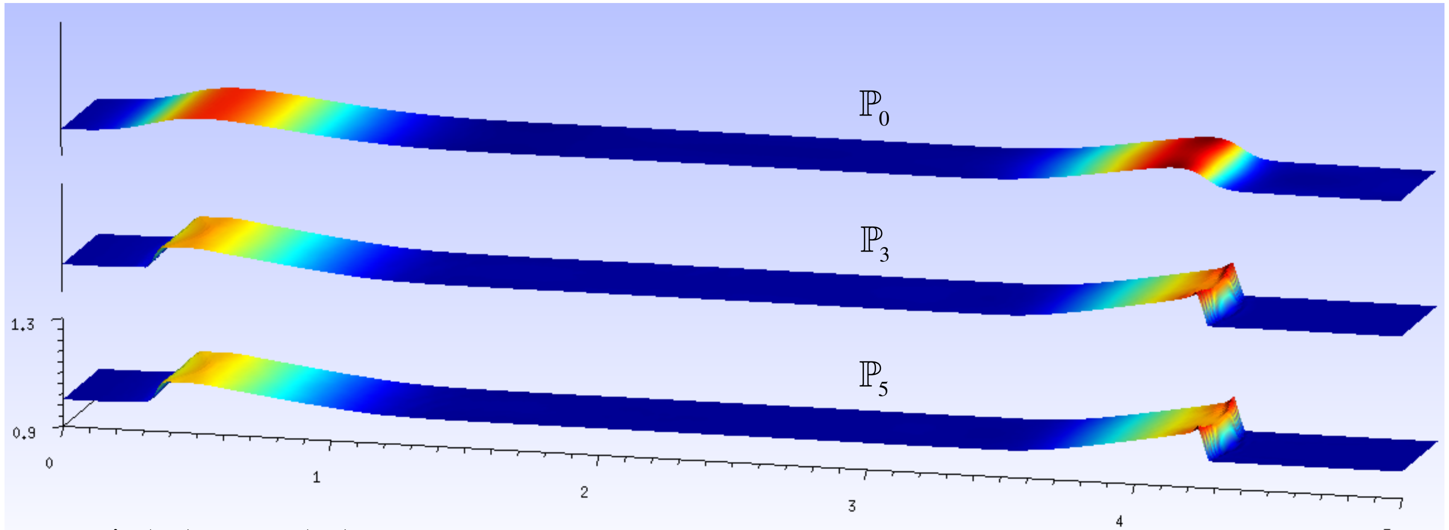
Purpose: check importance of high-order approx. to reduce numerical diffusion

Initial: $H = 1 + 0.2\exp\left(-20(x-2.5)^2\right), \quad \mathrm{U} = 0, \quad b = 0.2x\left(0.75 + 2(y-0.25)^2\right)$



- NS: Travelling wave (2)

Simulation: 16128 cells; final time: $t = 0.65$; b reconstruction with $\mathbb{P}_3$



	$\min(H)$	$\max(H)$
$\mathbb{P}_0$	0.999	1.081
$\mathbb{P}_1$	0.996	1.122
$\mathbb{P}_2$	0.994	1.124
$\mathbb{P}_3$	0.997	1.122
$\mathbb{P}_5$	0.998	1.128

- Conclusions

- By introducing an appropriate "correction" to the source term in the form of a non-conservative flux, we derived a new "well-balanced" scheme for the FV method based on the PRO-MOOD technique for the 2D SW problem.

- The "C-property" holds for the lake at rest, still preserving the (high-) order of convergence of the method.

- Numerical simulations done for several classical test cases confirm the high-order (up to h^6) of the method due to the use of the u2 detection process, where DMP technique alone goes no further than order 2.

- The use of a high-order scheme allows to better preserve the shape of travelling waves (less numerical diffusion).